Theory and Agent-Based Modeling of Taxpayer Preference and Behavior

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Abstract—**The relationship between tax rates and revenue guides governments in making policy decisions regarding tax reforms. The Laffer curve suggests the possibility of an inverse relationship between tax rates and revenue. There are conflicting views and opinions regarding this relationship and the shape of the Laffer curve. Since the behavioral responses of taxpayers to taxation is convoluted, with individuals having their own preferences, it is possible to use agent-based modeling. This paper creates an agent-based model that simulates taxpayer behavior to explore the relationship between tax rates and revenue. Two fundamental responses to taxation: alteration of labor supply, and tax evasion, are simulated in our agent-based model. The revenue collected for different tax rates is plotted and by doing so, the shape of the Laffer curve is verified. We estimate the unemployment rate for varying tax rates and explain the rationale behind this relationship, and further examine the direct and indirect effects of non-labor income taxes on revenue by plotting the revenue collected against labor income tax rates for different tax rates on non-labor income. To estimate the effect of audit rates on revenue, we analyse the percentage of revenue evaded for different audit rates, and finally numerically estimate the satisfaction of taxpayers at varying tax rates.**

Index Terms—**Agent-based modeling, computational economics, Laffer curve, labor supply, tax evasion**

I. INTRODUCTION

Tax reforms elicit different responses from taxpayers and have a direct bearing on the revenue collected by the government. There are several studies that affirm the taxpayer's responsiveness to taxation, particularly among high-income earners [1]–[3]. The Laffer curve [4] theorizes the relationship between tax rates and revenue and depicts the possibility of an inverse relationship between tax rates and revenue—the actions taken by taxpayers to reduce their tax liability causes a decrease in the revenue beyond a revenue-maximizing tax rate. Even so, increasing taxation is seen as a viable way to generate more revenue. In times of high deficits, when a government is required to generate more revenue, the ability to predict the effects of a given tax reform becomes critical. Hence, estimating the relationship between tax rates and revenue has important ramifications on the policy measures regarding taxation.

Despite the importance of the relationship between tax rates and revenue, no quantitative estimates of the Laffer curve using economic models have been made. However, there are several studies that create elasticity estimates and attempt to verify the presence of the Laffer curve, and thereby suggest the presence or absence of a revenue-maximizing tax rate beyond which revenue decreases. Kazman [5] presents a summary of these. Canto *et al.* [6] built a time series model to create elasticity estimates of revenue with respect to tax rates and found that in the years following 1964, decreasing tax rates could raise tax revenue, suggesting the presence of the Laffer curve. Lindsey [1], Feldstein [2] and Goolsbee *et al.* [7] used a difference-in-differences method to create elasticity estimates of revenue related to the top marginal income tax rate for high income earners. The estimates of elasticity calculated by Lindsey [1] and Feldstein [2] suggest the presence of the Laffer curve in the US during the years 1980–1984 and 1985–1988, while those calculated by Goolsbee *et al.* [7] do not indicate the presence of the Laffer curve in the US during the years 1922–1926, 1931–1938, 1948–1952 and 1962–1966. Apart from the approach of estimating elasticities, Ballard *et al.* [4] create a general equilibrium model that plots the relationship between tax rates and revenue for a variety of labor supply elasticities and outputs Laffer curves for different labor supply elasticities.

These earlier studies have dealt with estimating elasticities that suggest the presence of the Laffer curve but there have not been many studies that actually estimate the relationship between tax rates and revenue. In existing models it is also not possible to correlate tax rates with parameters excluding the revenue, such as unemployment rate and satisfaction of the taxpayers.

With the advent of agent-based modeling, it has become possible to simulate simultaneous actions of multiple autonomous agents to recreate and predict some complex phenomena, and hence it is fitting to use the same to model individual taxpayer actions. Simulating taxpayer behaviors using agent-based modeling does not only shed light on the relationship between tax rates and revenue collected, but also allows the analysis of other parameters such as unemployment rate and satisfaction.

We create an agent-based model that simulates two fundamental taxpayer responses among those identified by Giertz [8][1]: alteration of an individual's labor supply and tax evasion.

Tax reforms affect a taxpayer's decision regarding the consumption of goods and leisure, and this choice of consumption of goods versus leisure for individuals is modeled using the consumption-leisure model [9]. The optimal mix

[1]Giertz [8] presents four ways in which taxpayers respond to taxation: alteration of an individual's labor supply, tax evasion and avoidance, change in timing of income receipt, and response to administration and compliance policy.

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of consumption and leisure for an individual is derived in Theorem III.1, and the reservation wage for an individual is derived in Lemma III.2.

For simulating tax evasion behavior, an agent's decision to evade taxes proceeds according to Algorithm 1 which utilizes the direct effect of Bloomquist's model for tax evasion [10].

The agents in our model possess two kinds of attributes: base agent and derived agent attributes. The base agent attributes are the basic set of attributes that differentiate agents from each other, and the derived agent attributes represent some parameters that help in the modeling of agent responses to taxation, and therefore have direct influence over these agent responses. In addition to these attributes, the model also consists of a set of environment variables that also influence the responses from agents. Our agent-based model simulation proceeds in two phases: the setup and the repeat phase as presented in Algorithm 2. During the setup phase, the base agent attributes are assigned, and during the repeat phase, the derived agent attributes are calculated for each agent, following which auditing and tax collection are performed by the moderator.

The primary outcomes of our paper are as follows:

- *Study of the relationship between tax rates and revenue.* Our agent-based model outputs revenue collected at different labor income tax rates and thereby verifies the existence of the Laffer curve as seen in Figure 4. Note that the traditional Laffer curve assumes a flat tax rate on income from all sources, whereas our agent based model considers separate tax rates for labor and non-labor income (such as income from investments). For this reason, the curve depicting the relationship between tax rates and revenue output by the model is shifted upward (the point X in Figure 4 is above the origin) and the magnitude of this upward shift is dependent on the non-labor income of the taxpayers and the tax rate on this income.
- *Evaluation of the association between unemployment rate and labor income tax rates.* The model outputs the unemployment rate in the agent set for different labor income tax rates as depicted in Figure 5 and indicates how varying unemployment causes the relationship between labor income tax rates and revenue.
- *Analysis of the impact of non-labor income tax rates and audit rates on revenue.* The model estimates both the direct and indirect impact of non-labor income tax rates on revenue by plotting the revenue against labor income tax rates for various non-labor income tax rates as shown in Figure 6. Also, effects of tax evasion at different audit rates are estimated in the model by plotting the percentage of revenue evaded for different audit rates as seen in Figure 7.
- *Numerical estimation of the satisfaction that taxpayers derive from the consumption of goods and leisure at differ different labor income tax rates.* Since a government's goal is not only to maximize revenue, but also to ensure the welfare or satisfaction of taxpayers, being able to quantify the taxpayer's satisfaction is important. According to our model, the satisfaction that a taxpayer derives from the consumption of goods and leisure is represented by the utility. Hence, the satisfaction of the taxpayers at different tax rates is estimated by calculating the average utility of the taxpayers and is plotted for different labor income tax rates as depicted in Figure 8.

The remainder of the paper is structured as follows. Section II includes relevant theory that delineates concepts that form a basis for the model. In Section III, an economic model that builds on the concepts described in Section II is presented. It includes derivation of parameters that are used to model tax response behavior. Section IV describes the proposed agent-based model for simulating two fundamental behavioral responses to taxation—alteration of labor supply, and tax evasion. Section V presents the results output by the model and attempts to explain the tax-paying behavior. Finally, Section VI concludes the paper.

II. Relevant Theory

A. Consumption-Leisure Model

Kazman [5] suggests that altering labor supply is the most fundamental way an individual reacts to taxation. Hence, it becomes imperative to understand how an individual makes decisions regarding consumption and leisure. This is done by the consumption-leisure model and is therefore used to model the labor supply response of a taxpayer.

The time available to an individual can be spent either working or on leisure. Taxation affects an individual's choice regarding the consumption of goods or leisure. This choice explains the behavior of individuals with varying after-tax wages. For example, if the labor income tax rates are exorbitant, an individual prefers to consume less of goods and services and more of leisure. However, if the labor income tax rates are very low, the individual prefers consume more of goods and services and less of leisure. In this model, an individual receives satisfaction from both, the consumption of goods and services C as well as the consumption of leisure L. The variable C represents the total value of all the goods and services consumed by the individual and L represents the number of hours of leisure during the same time period. The concepts relevant to the consumption-leisure model that helps model the labor supply response is as follows.

1) Indifference Curves: The idea that an individual derives satisfaction from the consumption of goods and leisure can be represented in the form of a utility function [9] as follows.

$$u = U(L, C)$$

This utility function returns a utility index u that represents an individual's satisfaction that is derived from the consumption of goods and leisure. Different combinations of consumption of goods and leisure may yield the same utility. The locus of all points that yield equal values of utility is called an indifference curve. Hence, an individual's preference for consumption of goods and leisure can be modeled by choosing an indifference curve that fits their preference. These indifference curves are convex to the origin and downward sloping, and since both consumption of goods and leisure yield utility, indifference curves farther from the origin yield more utility than those closer to the origin.

In Figure 1, $\mathrm{I}_{\mathrm{P}}^{0}$, $\mathrm{I}_{\mathrm{P}}^{*}$ and $\mathrm{I}_{\mathrm{P}}^{1}$ represent a subset of indifference curves for an individual P. Since indifference curves farther from the origin yield more utility, $\mathrm{I}_{\mathrm{P}}^{1}$ would yield more utility than $\mathrm{I}_{\mathrm{P}}^{*}$ which in turn would yield more utility than $\mathrm{I}_{\mathrm{P}}^{0}$.

2) Budget Constraint: An individual's consumption of goods is constrained by their income, and the consumption of leisure is constrained by the amount of time available to the individual. The budget constraint is a line that represents the different combinations of the consumption of goods and leisure that is affordable by an individual [9]. A part of an individual's income may be independent of the number of hours worked and the wage rate, being earned from other sources such as property income and dividends. This income is called the non-labor income and is represented by V. Let h be the number of hours worked and w be the hourly wage rate. The consumption C is subject to a budget constraint as follows:

$$C = wh + V$$

If T is the total amount of time available in the period, the right hand side can be re-written as $w(T - L) + V$, which in turn gives

$$C = (wT + V) - wL \tag{1}$$

Hence, with L along the x-axis and C on the y-axis, the slope of the budget line should give $-w$ as depicted by the negative slope of line FE in Figure 1.

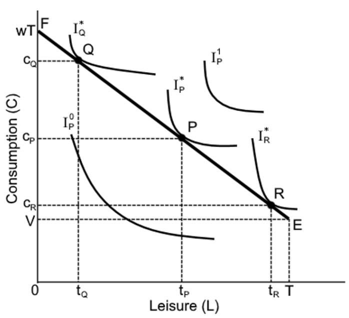

Fig. 1. Budget constraint and indifference curves

The combinations of consumption and leisure on or below the budget line FE in Figure 1 are viable to the individual, while those which lie above FE are not. In other words, the budget constraint represents the boundary of an individual's opportunity set—the set of consumption bundles the individual can afford to buy.

B. Tax Evasion Behavior

For modeling tax evasion behavior, our agent-based model uses Bloomquist's model [10]. Using this model, the simulation of a taxpayer's response to a change in audit rate, penalty rate, auditor efficacy, and enforcement celerity (the time between an act of evasion and its detection) can be achieved. According to this model, an individual decides to evade taxes if the following condition is satisfied:

$$1 - p - \frac{(p \times f \times d)}{(1 + r_d)^t} > 0 \tag{2}$$

where p is the perceived audit rate, f is the penalty rate, d is the detection rate, r_d is the discount rate, and t is time elapsed between the act of evasion and its detection.

Some reasons offered to account for the over-estimation are the guilt and shame associated with being caught evading [11] and over-estimation of low-probability events like audits [12]. To overcome this problem of over-estimation of audit rate, Bloomquist [10] assumes that the agents possess a perceived audit rate that is higher or lower than the actual audit rate.

III. Economic Model

A. Choice of Indifference Curve

In our model, the preferences of individuals are given by the Cobb-Douglas utility function [13] with constant returns to scale. Constant returns to scale indicate that a change in all input variables L and C causes a proportional change in the output utility u. The utility function is as follows:

$$U(L, C) = L^a C^{1-a} \qquad 0 \leq a \leq 1 \tag{3}$$

Here, a is the Cobb-Douglas constant which represents an individual's preference for leisure. Essentially, a represents the weight an individual gives to leisure, with $1 - a$ weight being given to consumption. A Cobb-Douglas preference constant of 0.5 implies that the individual has an equal preference for the consumption of goods and leisure. A Cobb-Douglas preference constant above 0.5 indicates that the individual prefers consumption of leisure over goods, and one less than 0.5 indicates that the individual prefers consumption of goods over leisure.

B. Optimal Consumption Bundle

Individuals make choices that maximize their satisfaction derived from consumption of goods and leisure. As indicated in Section II-A1, indifference curves that are farther from the origin yield more utility. The consumption bundle that maximizes the utility must lie on the indifference curve that is farthest from the origin and on the budget constraint line. Hence, the optimal consumption bundle would be the point at which the indifference curve is tangent to the budget constraint. This point of tangency that yields maximum utility $\mathrm{I}_{\mathrm{P}}^{*}$ that corresponds to the optimal consumption bundle $(c1, t1)$ is represented as point P in Figure 1. For an individual, the optimal consumption of goods C and leisure L can be given as follows.

Theorem III.1. *For an individual possessing a wage rate w, non-labor income V and Cobb-Douglas preference constant a and a total time T available for allocation between leisure* *and work, the optimal allocation for consumption of leisure and of goods is given by:*

$$L = aT + \frac{aV}{w} \tag{4}$$

$$C = (1-a)(wT+V) \tag{5}$$

Proof: The slope of the indifference curve is the negative ratio of the marginal utility obtained by consumption of leisure over the marginal utility obtained by consumption of goods.

$$\frac{dC}{dL} = -\frac{MU_L}{MU_C}$$

where MU_L is the marginal utility of the consumption of leisure and MU_C is the marginal utility of the consumption of goods. The marginal utilities of L and C are derived by finding the partial derivative of the utility function (3) with respect to L and C respectively. Equations (6) and (7) represent the marginal utilities of the consumption of leisure, and of the consumption of goods, respectively.

$$MU_L = \frac{\partial U}{\partial L} = aC^{1-a}L^{a-1} \tag{6}$$

$$MU_C = \frac{\partial U}{\partial C} = (1-a)C^{-a}L^{a} \tag{7}$$

The slope of the indifference curve is found by dividing the marginal utilities as represented below.

$$\frac{dC}{dL} = -\frac{aC}{(1-a)L}$$

At the point of tangency, slope of indifference curve is equal to slope of budget line. The slope of the budget line is $-w$ according to (1).

$$-w = -\frac{aC}{(1-a)L} \tag{8}$$

Solving for L and C from (1) and (8), we arrive at (4) and (5). ■

C. Reservation Wage

In addition to the selection of the optimal consumption bundle, each individual is found to have a minimum threshold wage, known as the reservation wage $\hat{w}$, below which the individual chooses not to work. The reservation wage for an individual can be given as follows8.

Lemma III.2. *For an individual possessing a non-labor income V, Cobb-Douglas preference constant a and a total time T available for allocation between leisure and work, the reservation wage $\hat{w}$ is given by:*

$$\hat{w} = \frac{aV}{(1-a)T} \tag{9}$$

Proof: The point on the budget line where an individual decides not enter the labor market, i.e., when $L = T$, is called the endowment point E [9] as depicted in Figure 1. For an individual whose wage is less than their reservation wage, the only source of income is the non-labor income V. This means that E corresponds to the consumption of V worth of goods and T hours of leisure. The slope of the indifference curve at this point E yields the minimum wage for which an agent enters the labor market. Substituting T and V for L and C in (8), we obtain (9). ■

It is now possible to calculate the optimal consumption bundle, reservation wage and utility for an individual with a labor wage w, non-labor income V and Cobb-Douglas constant a.

IV. Agent Based Model

Agents in the model are considered homogeneous workers in the labor market and are differentiated from one other based on the parameters that are assigned to model their responses.[2] Apart from the agents, the model includes a moderator which is representative of a government that is responsible for the collection of taxes and taxpayer auditing. The model supports simulations with different numbers of agents. Agents in the model are defined by their attributes and behaviors. In addition to attributes, there are environment variables which also influence the behaviors of agents. The following sections present the different agent attributes, behaviors and environment variables.

A. Attributes

The attributes of an agent include base and derived agent attributes.

Base Attributes: Base agent attributes are the basic set of non-derived attributes of an agent that differentiate agents and engender different responses from them. These attributes are summarized in Table I. These attributes are assigned to each agent during the setup phase of the model. The discussion surrounding the assignment of these variables is presented in Section IV-B. A brief description of the base attributes is as follows.

TABLE I
BASE AGENT ATTRIBUTES OF AGENT i.

Agent attribute	Symbol
Labor wage	w_i
Non-labor income	V_i
Perceived audit rate	p_i
Honesty	h_i
Discount rate	r_i

1) w_i is the labor wage earned by agent i per hour.
2) V_i is income that is not derived from the labor market by agent i in that year.
3) p_i represents agent i's perception of the actual audit rate.
4) h_i represents the honesty of agent i. If an agent decides to evade, then h_i represents the percentage of tax paid by agent i.
5) r_i represents the discount rate that an agent i applies to taxes and penalties that occur in the future.

Derived Attributes: The remaining attributes of an agent are the derived attributes presented in Table II.

[2]Agents in the model represent individual taxpayers in the real world, and hereon the two words “agent” and “individual” are used interchangeably.

TABLE II
DERIVED AGENT ATTRIBUTES OF AGENT i.

Agent attribute	Symbol
Reservation wage	$\hat{w}_i$
Cobb-Douglas preference constant	a_i
Labor wage after tax	w_i'
Non-labor income after tax	V_i'
Leisure	L_i
Consumption	C_i
Utility	U_i
Work	W_i
Annual labor income	Y_i
Evade	E_i
Detected	D_i
Time elapsed between evasion and detection	t_i
Last audit time	γ_i

A brief description of the derived attributes is presented below.

1) $\hat{w}_i$ is the reservation wage of agent i and is derived by applying (9).
2) a_i is Cobb-Douglas preference constant that defines the amount of preference an agent gives to consumption of leisure and goods.
3) w_i' is the after-tax hourly labor wage of agent i.
4) V_i' is the after-tax non-labor income of agent i.
5) L_i represents the number of hours of leisure that agent i chooses to take. Agent i calculates its optimal consumption of leisure according to (4).
6) C_i represents the optimal consumption of goods for agent i. Agent i calculates its optimal consumption according to (5).
7) U_i is the utility of agent i. It represents the satisfaction that agent i derives from the consumption of goods and leisure. Utility is calculated using (3).
8) W_i is the number of hours agent i has chosen to work in that year. W_i is the difference between the total number of hours available and number of leisure hours, i.e., $W_i = T_i - L_i$.
9) Y_i is the annual income from the labor market for agent i. It is the product between the number of work hours and hourly wage rate for agent i.
10) E_i is a boolean attribute that is set true if agent i makes a decision to evade taxes that year. It is set false during setup.
11) D_i is a boolean attribute that is set true if agent i is caught evading taxes during auditing. It is set false during setup.
12) t_i represents the time that has elapsed between evasion and detection for agent i.
13) γ_i represents the time elapsed after the last audit for agent i.

Environment Variables: Apart from agent attributes, the model includes a set of environment variables which also influence the responses of agents. These variables are listed in Table III.

A brief description of the environment variables are presented below.

TABLE III
ENVIRONMENT VARIABLES

Agent attribute	Symbol
Labor tax rate	α
Non-labor income tax rate	β
Total time	T
Audit rate	ϵ
Penalty rate	f
Detection rate	d
Discount rate mean	μ_{rd}
Discount rate standard deviation	σ_{rd}
Detected time effect	δ

1) α is the flat tax rate levied the labor wages of agents.
2) β is the flat tax rate levied on the non-labor income of agents.
3) T is the total number of hours in a year available for allocation between leisure and work for agents. Assuming 5 working days a week and 16 hours of active time a day, the total time amounts to 4160 hours a year that is left for allocation between work and leisure.
4) ϵ is rate at which auditing is conducted by the moderator to detect tax evaders.
5) f is the penalty rate that a tax evader is charged if caught during auditing.
6) d represents the rate at which a tax evader is caught if audited by the moderator.
7) μ_{rd} represents the mean of the discount rate for the normal distribution of discount rate in agents.
8) σ_{rd} represents the standard deviation of the discount rate for the normal distribution of discount rate in agents.
9) δ represents the number of rounds that an agent takes to consider evading taxes again after being caught evading during auditing.

B. Agent behaviors

Labor Supply Response: During every round in the simulation, each agent calculates the optimal consumption bundle based on the parameters assigned and the tax structure. After the selection of the optimal consumption bundle, the annual income for all agents from the labor market is known, and tax collection is carried out by the moderator (the government responsible for tax collection and auditing) as depicted in Algorithm 2 (line 31).

The agent attributes that have a direct bearing on the labor supply response are labor wage w_i, non-labor income V_i and Cobb-Douglas preference constant a (see (4) and (5)). The assignment of these attributes is discussed as follows. Heckman and Sattinger [14] explain why the labor wage distribution (exclusive of the capital income) would follow a log-normal distribution. The log-normal distribution is therefore used by us to model the labor wages of agents. Nedved [15] used log-normal distribution models and estimated the parameters for the distribution that best fits the 2010 Czech Republic hourly wage data and concluded that the 3-parameter log-normal distribution fits that data well.

The parameters estimated by Nedved [15] are used by us to model the hourly wage of agents according to a 3-parameter log-normal distribution whose probability density function is represented as follows.

$$\mathrm{pdf}(x) = \frac{e^{\frac{-(ln(x-\theta)-\mu)^2}{2\sigma^2}}}{(x-\theta)\sigma\sqrt{2\pi}} \qquad x > 0, \sigma > 0 \tag{10}$$

After fitting the mean μ, standard deviation σ and threshold θ parameters to be 4.6256, 0.5933 and 22.3968 as estimated by Nedved [15], the probability density function for wages was plotted and is as shown in Figure 2. Figure 2 represents

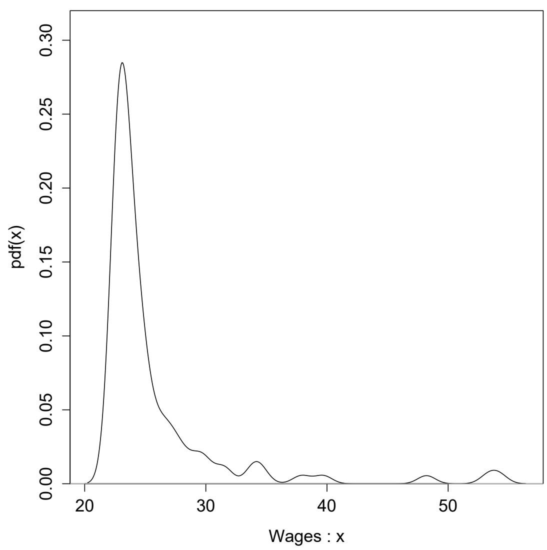

Fig. 2. Probability density function of wages in agents

the density of agents at different hourly wage rates. However, there seem not to have been studies of the distribution of non-labor income. Piketty and Goldhammer [16] suggest that the distribution of non-labor income is more unequally distributed than labor income. For this reason, a highly skewed log-normal distribution is used to model the non-labor income. The agents are assigned their labor wage w_i and non-labor income V_i attributes during the setup phase of the simulation as seen in Algorithm 2.

An agent's Cobb-Douglas preference constant a_i is closely connected with the non-labor income V_i. The assignment of an agent's Cobb-Douglas preference constant is based on the argument that an agent with a lower non-labor income has more incentive to work, as the agent is left with no other options to make ends meet. On the other hand, an agent with high non-labor income has less incentive to work since the agent's necessities would be met even without income from the labor market.

The agent's non-labor income after taxes V_i' is scaled down to the range [0.3,0.7] and assigned as the Cobb-Douglas preference constant a_i (line 18). Agents with Cobb-Douglas preference constant values below 0.3 choose to stay in the labor market even for exorbitant labor income tax rates (even 100%) and agents with values greater than 0.7 leave the labor market even for negligible labor income tax rates. Hence, the range [0.3,0.7] is chosen because inclusion of agents with preferences outside this range is not representative of the real world. The assignment of an agent's Cobb-Douglas preference constant is shown in Figure 3. For an agent with less incentive

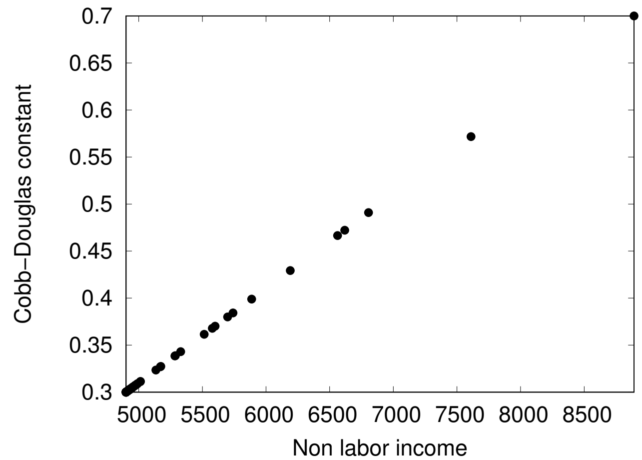

Fig. 3. Cobb-Douglas preference constant for different non-labor incomes of agents

to work, the indifference curve is steep, indicative of the fact that the agent requires a large consumption of goods to give up any leisure. For an agent with more incentive to work, the indifference curve is flat, indicating that the agent requires less consumption of goods to give up some leisure. The incentive to work is reflected in the choice of the optimal consumption bundle. For the same budget constraint, an agent with less incentive to work chooses more leisure hours than working hours, and an agent with more incentive to work chooses more working hours than leisure hours, as illustrated in Figure 1. In Figure 1, $\mathrm{I}_{\mathrm{Q}}^*$ and $\mathrm{I}_{\mathrm{R}}^*$ represent the indifference curves for two individuals Q and R that yield maximum utility. The indifference curve $\mathrm{I}_{\mathrm{R}}^*$ is steep and $\mathrm{I}_{\mathrm{Q}}^*$ is flat relative to $\mathrm{I}_{\mathrm{P}}^*$. As seen in the figure, for the same budget constraint FE, the individual Q whose indifference curve is flat chooses more consumption of goods and less leisure hours compared to individual P. On the other hand, individual R whose indifference curve is steep chooses less consumption of goods and more leisure hours compared to individual P.

In addition to the consumption-leisure trade-off decision, an agent's Cobb-Douglas constant also influences the reservation wage $\hat{w}_i$. An agent with more preference for leisure has a higher reservation wage. This means that for two agents with the same non-labor income, with one agent having more preference for leisure than the other, the minimum wage required to galvanize the agent with the higher preference for leisure is higher than for an agent with the lower preference for leisure. In this manner, preferences for consumption of goods and leisure are modeled in agents. Hence, the agent chooses its optimal consumption bundle based on labor wage, non-labor income, Cobb-Douglas constant and the tax rates on labor and non-labor income currently set in the environment.[3]

Tax Evasion Behavior: In our model, direct effects of the Bloomquist's model [10] of tax evasion behavior are simulated.

According to Bloomquist's model of tax evasion behavior [10], agents declare a certain percentage of their income to be visible (income reported to the tax agencies and therefore taxed). The concept of visible income was introduced to represent institutional arrangements that positively influence the level of reporting compliance. The labor income Y_i in our agent based model is visible income as most institutions have arrangements that ensure reporting compliance. Christian [18] suggests that taxpayers report 99% of their visible income and Thomas *et al.* [19] suggest that taxpayers report only 67% of non-labor income. This data suggests that taxpayers decide to pay a certain percentage of taxes on non-labor income. In our agent-based model, this percentage of taxes on non-labor income that an agent decides to pay is defined as the agent's honesty.

The agent attributes that influence the tax evasion response are honesty, perceived audit rate and discount rate which are assigned during the setup phase of the simulation as seen in Algorithm 2 (lines 8–10).

If the agent decides to evade, the amount of tax paid increases proportionately with the agent's honesty. To encompass a diversity of agents with different levels of honesty, our agent-based model assumes a normal distribution for honesty in agents with a mean of 40 and a standard deviation of 10, i.e., in Algorithm 2 (line 9), μ_3 and σ_3 are assigned as 40 and 10 respectively.

After taxes are collected by the moderator, auditing is done to detect tax evaders. Audit rate in the US during the year 2014 was 0.85% and has been declining in the following years. In the year 2017, the audit rate was found to be 0.6% and the same is assumed by our model for simulations [20]. As mentioned previously in Section II-B, the over-estimation of audit rate is settled by inclusion of a perceived audit rate in the agents. Bloomquist's model [10] assigns a perceived audit rate p_i to each agent according to a normal distribution with mean as the actual audit rate ϵ and standard deviation of 3%. Simulations in Bloomquist's model [10] assume a penalty rate of three times the amount of taxes evaded and the discount rates in agents to be normally distributed with mean μ_{rd} and standard deviation σ_{rd}. Our agent based model assumes similar distributions for perceived audit rate, penalty rate and discount rate in agents.

The algorithm that decides whether an agent i evades taxes or not is presented in Algorithm 1. At every round, each agent updates the last evaded time (line 3). Integrated with the auditing function, if an agent is audited, it updates the last audited time or else this parameter is by default assigned to 0. If the agent has been caught evading taxes, then it waits for a period of δ rounds before attempting to evade again (lines 4–6). If the agent is not caught, then (2) is evaluated and the evasion decision is made (line 8).

[3]There does not seem to be a quantitative theory to account for the fact that individuals may be motivated to work harder when they see clear rewards, and that workers with higher talent often get better wages—up to a point. A "wage-talent curve" [17] and its associated factors would add further nuances to the labor supply response behavior of agents.

Algorithm 1: Evasion Decision

1 **foreach** *agent i* **do**
2   **if** *Evade=True* **then**
3         $t_i \leftarrow t_i + 1$
4   **if** D_i=*True* **then**
5   **if** γ_i=δ **then**
6             $D_i \leftarrow False$
7   **else**
8   **if** $1 - p_i - \frac{(p_i \times f \times d)}{(1+r_i)^{t_i}} > 0$ **then** // see (2)
9   return True
10   **else**
11   return False
12 **end**

C. Simulation of Taxpayer Behavior

Implemented in the NetLogo simulation tool, the simulation is run with hundred agents and proceeds as presented in Algorithm 2. The simulation consists of a setup phase and a repeat phase which is performed indefinitely each round. In the setup phase (lines 2–3, 6–7), the labor wage w_i and non-labor income V_i of an agent are assigned according to a 3-parameter log-normal distribution as represented in (10). The distribution of labor wages w_i in agents is as depicted in Figure 2. Also, during the setup phase the other base agent attributes are assigned according to distributions mentioned earlier this section. During the repeat phase, an agent's labor and non-labor income after taxes are updated and the reservation wage $\hat{w_i}$ is calculated (lines 17–19). If the agent's hourly wage rate after taxes w_i' is less than the agent's reservation wage $\hat{w_i}$, then the agent decides not to work, and hence leisure hours L_i is set to the total time available T (line 21). Else, L_i is calculated using (4) (line 23). Following this, an agent's Cobb-Douglas preference constant a_i, consumption of goods C_i, work hours W_i, utility U_i and annual labor income Y_l are calculated (lines 24–28). As described earlier this section, the assignment of an agent's Cobb-Douglas preference constant a_i is done by scaling down the agent's non-labor income after taxes V_i' to the range [0.3,0.7] (line 24). Also, in the repeat phase the agents make their evasion decision, while the moderator performs revenue collection and auditing. During revenue collection, the agent's annual labor income and non-labor income(if the agent does not evade taxes) are taxed as usual. If the agent decided to evade taxes, then the revenue collected is percent of honesty h_i times the non-labor income after taxes V_i' of the agent. The moderator performs auditing randomly according to the audit rate and the success with which evasion is detected after auditing agent i depends on the detection rate. If an agent is audited, then the last audit time γ_i is set to 0 and if an agent

Algorithm 2: Simulation

Create specified number of agents (n). `// Setup phase`
$List\ income_L(n) \leftarrow lognormal(\mu_1, \sigma_1, \theta_1, n)$ `// see (10)`
$List\ income_{NL}(n) \leftarrow lognormal(\mu_2, \sigma_2, \theta_2, n)$ `// see (10)`
$income'_{NL}(n) \leftarrow (1 - \beta/100)^*(income_{NL}(n))$
foreach *agent i* **do**
  $w_i \leftarrow income_L(i)$
  $V_i \leftarrow income_{NL}(i)$
  $p_i \leftarrow \epsilon + normal(\epsilon, 0.03)$
  $h_i \leftarrow normal(\mu_3, \sigma_3)$
  $r_i \leftarrow normal(\mu_{rd}, \sigma_{rd})$
end
Loop
  $Revenue \leftarrow 0$ `// Repeat phase`
  foreach *agent i* **do**
    $min_{NL} \leftarrow min(income'_{NL}(n))$
    $max_{NL} \leftarrow max(income'_{NL}(n))$
    $V'_i \leftarrow (1 - \frac{\beta}{100}) \cdot V_i$
    $w'_i \leftarrow (1 - \frac{\alpha}{100}) \cdot w_i$
    $\hat{w_i} \leftarrow \frac{a_i}{(1-a_i)} \cdot (\frac{V'_i}{T})$ `// see (9)`
    if $w'_i < \hat{w_i}$ **then**
      $L_i \leftarrow T$
    else
      $L_i \leftarrow (a_i \cdot T) + \frac{(a_i \cdot V'_i)}{w'_i}$ `// see (4)`
    $a_i \leftarrow 0.3 + \frac{(V'_i - min_{NL}) \cdot 0.4}{(max_{NL} - min_{NL})}$
    $C_i \leftarrow (1 - a_i) \cdot ((w'_i \cdot T) + V'_i)$ `// see (5)`
    $W_i \leftarrow T - L_i$
    $U_i \leftarrow (L_i^a) \cdot (C_i^{1-a})$ `// see (3)`
    $Y_i \leftarrow w_i \cdot W$
    $E_i \leftarrow evasionDecision(i)$
  end
  `collectRevenue()`
  `auditing()`
EndLoop

is caught evading, then the boolean detected is set and the last evaded time t_i is set as 0.

V. Results and Discussion

A. The Laffer Curve

The traditional Laffer curve suggests the presence of a point between the 0% and 100% tax rates that maximizes tax revenues. Fullerton [21] estimates revenue-maximizing tax rates and suggests the mid range to be at 70%.

To verify the relationship suggested by the Laffer curve, we plot revenue collected against labor income tax rates for every 0.1% increment in labor income tax rates. Assuming that non-labor income is taxed at 30% and a log-normal distribution exists for labor and non-labor income, the model predicts the relationship depicted in Figure 4. In Figure 4, the model

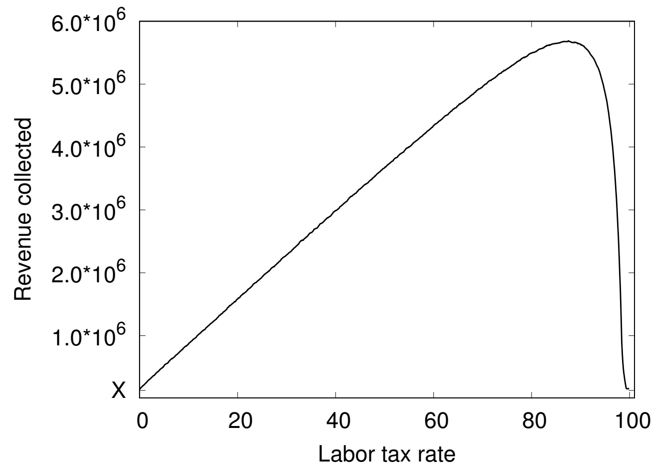

Fig. 4. The Laffer curve: Relationship between tax rates and revenue

outputs an asymmetric Laffer curve with revenue maximized at just over 85% labor income tax rate. The revenue at 0% and 100% labor income tax rate in the traditional Laffer curve is 0 since it assumes a single flat taxation on income. However, as labor and non-labor income are taxed separately, some revenue is contributed by the taxes on non-labor income and hence, the revenue collected at 0% and 100% labor income tax rates are non-zero as depicted by point X on the y-axis in Figure 4.

B. Unemployment Rate

The downward slope observed in Figure 4 is due to the fact that at higher rates of taxation, the taxpayers decide not to participate in the labor market. Observing the unemployment rate at different labor income tax rates can provide an explanation of the downward slope observed in Figure 4. Figure 5 is a plot of the tax rate versus unemployment rate. At

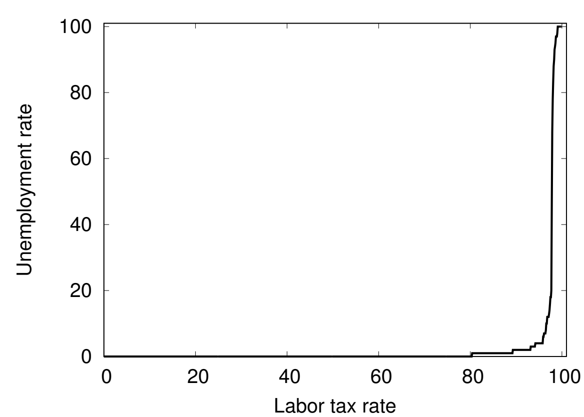

Fig. 5. Unemployment rate for different labor income tax rates

higher rates of taxation, the taxpayer's labor wage goes below the reservation wage, forcing them out of the labor market and hence increasing the unemployment rate. The increase in unemployment rate is commensurate with the decrease in revenue, offering a reasonable explanation as to why Figure 4 slopes downwards. Also, it can be observed that till about 85% labor income tax rate, the working population remains at 100%, indicative of the increase in revenue since all the workers pay accruing taxes till this point.[4] Beyond this point, even though the labor income tax rates are increasing, the revenue collected decreases, as the revenue lost due to agents not working is higher than the revenue increase due to the higher labor income tax rates.

C. Impact of Non-Labor Income Tax on Revenue

In Figure 4, the tax liability of non-labor income seems negligible compared to that of labor income. To observe the effect of non-labor income tax on revenue, revenue collected is plotted for five different rates of non-labor income tax keeping the labor wage distribution constant in Figure 6. As seen in

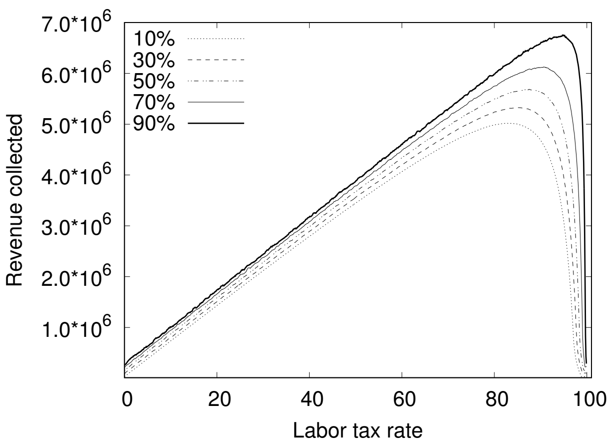

Fig. 6. Shift in inversion point with varying tax on non-labor income

Figure 6, there a clear shift of the inversion point to the right when the non-labor income tax rate is increased. For higher non-labor income tax rates, each agent lowers their reservation wage correspondingly, and works even with a higher tax rate on labor income. However, for low non-labor income tax rates, an agent's reservation wage rises, and the agent stops working at a relatively lower labor income tax rate. Although the tax liability of labor income appears far greater than that of non-labor income, increasing the non-labor income tax rate forces an agent out of the labor market at a relatively higher labor income tax rate. This means that the non-labor income tax rate's direct effect on the revenue is minor, but the magnitude of increase in revenue due to its effect on the reservation wage is large. Taking this into consideration, a government can increase revenue by increasing taxes on non-labor income, keeping more of the working population in the labor market for higher labor income tax rates.

D. Effect of Audit Rates

The model simulates the direct effects of tax evasion. To understand the implications of taxpayer auditing, it is reasonable to observe the fraction of revenue that is evaded for different audit rates. Note that only after an agent decides to evade taxes, the honesty of the agent decides how much of the taxes the agent will pay. Holding a constant 20% tax rate on labor and non-labor income, Figure 7 depicts the percentage of revenue evaded at different audit rates. A

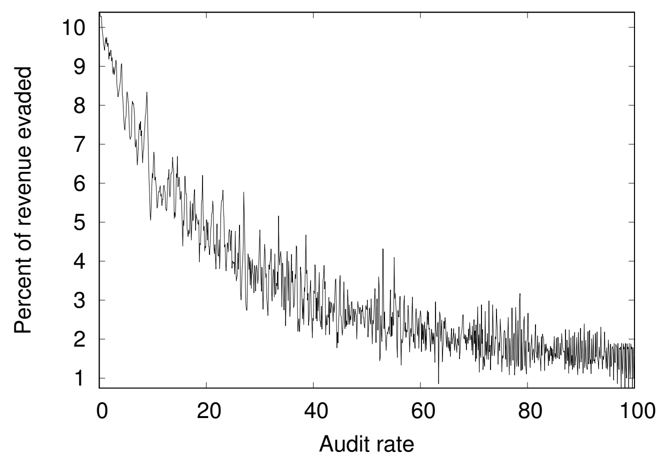

Fig. 7. Percentage of revenue evaded at different audit rates

downward trend is observed in Figure 7. About 10% of the revenue is evaded when no audits are conducted. A decreasing percentage of revenue is evaded for increasing audit rates, indicating that fewer agents evade taxes when audits are conducted vigorously. This indicates, as is intuitive, that a practical solution for the government to increase revenue is to increase the audit rate.

E. Population Satisfaction

The government's goal is not only to maximize revenue, but to also ensure the welfare or satisfaction of the taxpayers [22]. Discerning the satisfaction of the taxpayers at different labor income tax rates can possibly assist tax agencies in devising tax policies. The satisfaction of the agents in our model is reflected by their utility. Figure 8 portrays the satisfaction of the taxpayers by plotting the average utility of the taxpayers at different labor income tax rates. Figure 8 suggests that on an average the population's satisfaction decreases almost linearly till a high labor income tax rate. At a labor income tax rate above 95%, the decrease is even more rapid. It can be inferred that the population becomes decreasingly satisfied for increasing labor income tax rates. Hence, while devising a tax structure, the tax agency should find a trade-off between the revenue collected and the satisfaction of the population that gives sufficient revenue and also satisfies the tax payers.

[4] In our analysis, we make the assumption that any individual who wishes for employment can get it; this is obviously a simplification as in reality, individuals may not find employment due to lack of skills, education, suitable opportunities, or other barriers. Likewise, we assume that the labor income being above the reservation wage is the sole criterion for an individual to remain employed, which is also a simplification, as in reality an individual can cease employment because of other reasons such as ill health, job dissatisfaction, or family issues.

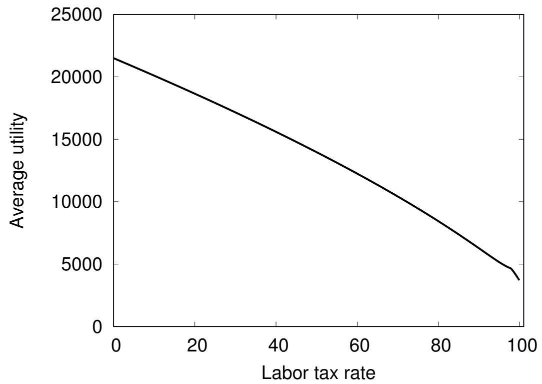

Fig. 8. Population's satisfaction on an average from the consumption of goods and leisure at different labor income tax rates

VI. Conclusion

The agent based model simulates the taxpayer's fundamental responses to taxation, estimates the revenue collected and presents various trends that emerge as a result of the taxpayer behavior. The model outputs an asymmetrical Laffer curve with minor variations from the typical Laffer curve, thus verifying the presence of an inverse relationship between tax rates and revenue beyond a revenue-maximizing labor income tax rate. The model estimates the unemployment rate at different tax rates and demonstrates the correlation between the revenue collected and unemployment rate. By depicting the direct and indirect impacts of taxes on non-labor income, the model portrays how a government's decision to increase non-labor income tax rates can have a more profound impact on revenue than what was anticipated. The effect of audit rates on revenue is assessed in the model by estimating the percentage of revenue evaded at different audit rates. Finally, the model depicts the satisfaction the taxpayers derive from their consumption of goods and leisure at varying labor tax rates. The model can be calibrated with different configurations for the parameters used to model the responses to taxation. The configuration of parameters must most closely reflect empirical data for accurate predictions. With proper calibration, the model can assist in evaluation and decision-making regarding tax policies. Possible future work can extend the model to incorporate a marginal tax structure, and to consider the effect of tax rates on the tax evasion response of taxpayers. A development of a quantitative theory correlating wages and talent [17], or wages and effort or performance, is also needed. When such a theory is available, our agent-based model can be extended to include the additional factor of agents choosing to exert themselves to achieve a higher performance—or not—based on their liking for the corresponding rewards. Also, the model can be extended to encompass other responses to taxation, such as changes in timing of income receipt and tax avoidance.

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